

Domain Walls and Superpotentials from M Theory on Calabi-Yau Three-Folds

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Abstract

Compactification of M theory in the presence of G -fluxes yields $\mathcal{N} = 2$ five-dimensional gauged supergravity with a potential that lifts all supersymmetric vacua. We derive the effective superpotential directly from the Kaluza-Klein reduction of the eleven-dimensional action on a Calabi-Yau three-fold and compare it with the superpotential obtained by means of calibrations. We discuss an explicit domain wall solution, which represents five-branes wrapped over holomorphic cycles. This solution has a “running volume” and we comment on the possibility that quantum corrections provide a lower bound allowing for an AdS_5 vacuum of the 5-dimensional supergravity.

1. Introduction

In this paper we study compactification of M theory on Calabi-Yau three-folds in the presence of background G -fluxes. If there were no G -fluxes, the effective field theory would be $\mathcal{N} = 2$ five-dimensional supergravity interacting with some number of hypermultiplets and vector multiplets whose scalar fields parametrize a manifold \mathcal{M} . Turning on a non-trivial G -flux generates effective superpotential in the five-dimensional theory, which is related to gauging of global isometries of the scalar manifold \mathcal{M} . If the potential in the five-dimensional theory allows for isolated extrema, the vacuum is given by a space-time of constant negative curvature (i.e. an AdS space) and such a theory is relevant to the AdS/CFT correspondence [1]. On the other hand, we find that the potential is a monotonic function of volume scalar, and this “run-away” case is relevant to the generalization of the AdS/CFT correspondence, the so-called domain wall/QFT correspondence [2,3]. However, if the Calabi-Yau space has positive Euler number then the running of the volume is bounded by quantum corrections [4], so that five-dimensional supergravity has an AdS₅ vacuum. Below we list various applications which motivated our work.

Over the past year, domain walls as solutions of 5-dimensional gauged supergravity that interpolate between different vacua has been a subject of intensive research; for earlier work on domain wall solution of 4-dimensional supergravity see [5]. Most of them are dealing with the maximal supersymmetric case like in [6,7] and many subsequent papers, but also the least supersymmetric case has been discussed [8,9,10,11]. For a recent review see [12] and a discussion that of a running breathing mode is given in [13].

A model of domain wall universe was used by Randall and Sundrum [14] to address the hierarchy problem in a novel way, alternative to compactification. An interesting feature of Randall-Sundrum construction is that gravity is localized on the domain wall (or D-brane) by a suitable gravitational potential. The original construction of [14] is purely classical and is based on a non-supersymmetric example of a domain wall which interpolates between two regions of five-dimensional space-time with negative cosmological constant. However, locally AdS form of the vacua on each side of the wall suggests that there must be a corresponding supersymmetric solution. For a related recent work see [15]. Furthermore, motivated by the celebrated D-brane construction of MQCD [16], one would like to embed a model a la Randall-Sundrum in string theory or M theory to learn about non-perturbative effects in the theory on the domain wall. Even though we will not be able to solve this problem in the full generality, we hope that our study of domain

walls constructed from M5-branes wrapped on holomorphic curves inside a Calabi-Yau space will be a useful step in this direction. This configuration has been discussed in the heterotic M-theory compactification in [8].

Another line of research which motivated this paper is a quest for new supersymmetric vacua in compactifications of string theory or M theory on Calabi-Yau manifolds with background fluxes. Since the flux has to be quantized, its different values correspond to distinct disconnected components in the space of supersymmetric vacua. Therefore, if we call \mathcal{F} the background flux and $\mathcal{M}_{\mathcal{F}}$ the corresponding component of the moduli space, the total space of vacua looks like:

$$\mathcal{M} = \coprod_{\mathcal{F}} \mathcal{M}_{\mathcal{F}} \quad (1.1)$$

The component \mathcal{M}_0 is equivalent, at least locally, to the moduli space, $\mathcal{M}(Y)$, of the Calabi-Yau space Y . The other components are isomorphic to some subspaces in the Calabi-Yau moduli space, $\mathcal{M}_{\mathcal{F} \neq 0} \subseteq \mathcal{M}(Y)$, such that all points in $\mathcal{M}_{\mathcal{F}}(Y)$ correspond to the values of Calabi-Yau moduli which lead to supersymmetric compactifications on Y with a given flux \mathcal{F} . For example, when Y is a Calabi-Yau three-fold new supersymmetric vacua can be found at some special (conifold) points of the moduli space [17,18]. For a Calabi-Yau four-fold there is usually more possibility to turn on background fluxes which do not break supersymmetry further [19,20,21,22]. Since the value of the flux \mathcal{F} jumps across a brane of the appropriate dimension, this brane wrapped over a supersymmetric cycle in Y can be identified with a BPS domain wall interpolating between different components in (1.1). This interpretation was used in [20,22] to deduce the effective superpotential $W(\mathcal{F})$ generated by a flux \mathcal{F} in compactification on a Calabi-Yau four-fold Y , such that its minima over $\mathcal{M}(Y)$ reproduce the space of vacua $\mathcal{M}_{\mathcal{F}}$. Using a more general argument which also applies to compactifications on G_2 and $Spin(7)$ manifolds, one finds the following universal formula for the effective superpotential in terms of calibrations of Y [22]:

$$W = \sum \int_Y (\text{fluxes}) \wedge (\text{calibrations}) \quad (1.2)$$

The paper is organized as follows. In the next section we perform a Kaluza-Klein reduction of the eleven-dimensional supergravity action on a Calabi-Yau three-fold with a G -flux. Among other things we find that all supersymmetric vacua of the five-dimensional theory are lifted by the effective superpotential which does not have stable minima. In section 3 we rederive the same result identifying BPS domain walls with five-branes wrapped

over holomorphic curves, and argue that the formula (1.2) can be also applied to compactifications on Calabi-Yau three-folds. In section 4 we explicitly construct domain wall solutions in the effective $D = 5$ $\mathcal{N} = 2$ gauged supergravity which correspond to M5-branes wrapped over holomorphic curves in the Calabi-Yau space. The discussion in section 2 and 4 is in part parallel to the work [8], which we extend by the inclusion of quantum corrections yielding an AdS vacuum solution.

2. Compactification of M Theory on Calabi-Yau Three-Folds with G -Fluxes

In this section we perform the compactification of M theory on a Calabi-Yau three-fold Y with a G -flux. For the Kaluza-Klein reduction of the eleven-dimensional action:

$$S_{11} = \frac{1}{2} \int d^{11}x \sqrt{-g} R - \frac{1}{2} \int \left[\frac{1}{2} G \wedge *G + \frac{1}{6} C \wedge G \wedge G \right] \quad (2.1)$$

we follow the standard procedure, which lead to gauged $\mathcal{N} = 2$ supergravity in five dimensions as discussed in [8,9]. The latter theory has a potential for the scalar fields X^I that play the role of local coordinates on the moduli space of Kähler deformations of Y . Unfortunately, the scalar potential always exhibits a run-away behavior, so that compactification of M theory on Y with non-zero G -flux does not lead to new vacua in the effective five-dimensional theory. It is worth mentioning that most of the material in this section is not new and has appeared in the literature in various form. In particular, we follow the steps of [23] where analogous compactification on Calabi-Yau three-folds without G -fluxes was studied. In the context of Type II string theory compactifications with background fluxes were discussed in the work [17] where similar results were found. Closer to the subject of our paper is the work [8] where compactification of M theory with a G -flux was investigated and the induced superpotential was derived. In order to make the paper self-consistent, below we perform once again all the steps of the Kaluza-Klein reduction in the form that will be convenient later.

The Kaluza-Klein reduction on a Calabi-Yau three-fold Y yields $h^{1,1}$ abelian gauge fields entering $h^{1,1} - 1$ vector multiplets and a gravity multiplet [23]. The vector fields come from the light modes of the 3-form field C in eleven dimensions. Namely, for the field strengths we have a decomposition:

$$G \sim dA^I \wedge \omega_I \quad (2.2)$$

where $\omega_I \in H^{(1,1)}(Y)$ is a basis of $(1,1)$ -forms. Each vector multiplet contains besides the gaugino a real scalar which comes from the reduction of the internal metric $g_{a\bar{b}} = -i t^I (\omega_I)_{a\bar{b}}$, where t^I are the Kähler moduli. Identifying expectation values of the vector multiplet scalar fields with t^I , we can write the Kähler form as follows:

$$\mathcal{K} = t^I \omega_I \quad (2.3)$$

As we will see below, the scalar parameterizing the volume of the Calabi-Yau decouples from the vector multiplets and enters the universal hypermultiplet. This volume scalar is defined by:

$$\mathcal{V} = \int \sqrt{g_Y} = \frac{1}{3!} \int \mathcal{K} \wedge \mathcal{K} \wedge \mathcal{K} = \frac{1}{6} C_{IJK} t^I t^J t^K$$

and the scalars ϕ^A ($A = 1 \dots h^{1,1} - 1$) entering the vector multiplets are obtained from

$$1 = \frac{1}{6} C_{IJK} X^I X^J X^K \quad \text{with} \quad t^I = \mathcal{V}^{\frac{1}{3}} X^I \quad (2.4)$$

i.e. $X^I = X^I(\phi^A)$. In what follows we denote \mathcal{M} the manifold parameterized by the scalar fields ϕ^A , see figure below.

In addition to the volume scalar the universal hypermultiplet contains a real scalar which is dual to the 4-form field in 5 dimensions:

$$G \sim dC_3 \sim {}^* da \quad (2.5)$$

and a complex scalar coming from:

$$G \sim dm \wedge \Omega + d\bar{m} \wedge \bar{\Omega} \quad (2.6)$$

In addition to these scalars further scalars are related to non-trivial elements of $H^{(2,1)}(Y)$, which build up the remaining hyper multiplets. These fields are not important for our analysis, so, we will ignore them.

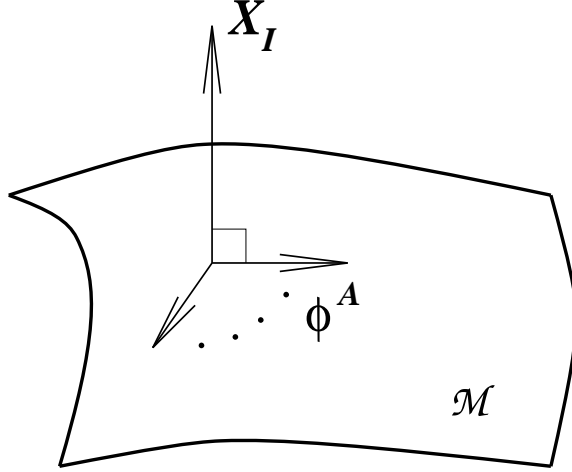


Fig. 1: Scalar components ϕ^A of vector multiplets parametrize the space \mathcal{M} defined by the hypersurface equation (2.4). At extrema of W , the normal vector X_I has to be parallel to the flux vector α_I .

In order to obtain the canonical Einstein-Hilbert term in 5d, we have to perform a Weyl rescaling (Einstein versus string frame) which is related to the volume of the internal space. In five dimensions this rescaling is given by:

$$ds_E^2 = \mathcal{V}^{\frac{2}{3}} ds_{str}^2 \quad , \quad \sqrt{g_{str}} = \sqrt{g_E} \mathcal{V}^{-\frac{5}{3}} .$$

with the eleven-dimensional metric written as $ds_{11}^2 = ds_{str}^2 + ds_{CY}^2$. Combining this rescaling with the rescaling of the scalars in (2.4), the reduction of the Ricci scalar yields [23]:

$$S_5 = \int \left[\frac{1}{2} R - \frac{1}{2} G_{IJ}(X) \partial X^I \partial X^J - \frac{1}{2} \frac{\partial \mathcal{V} \partial \mathcal{V}}{\mathcal{V}^2} \right]$$

where one has to use the relation $G_{IJ}(X) X^I \partial X^J = 0$ and G_{IJ} as function of the t^I coordinates is defined by:

$$G_{IJ}(t) = \frac{i}{2\mathcal{V}} \int \omega_I \wedge {}^* \omega_J = -\frac{1}{2} \left[\frac{C_{IJK} t^K}{\mathcal{V}} - \frac{1}{4} \frac{(C_{IKL} t^K t^L)(C_{JMN} t^M t^N)}{\mathcal{V}^2} \right]$$

After rescaling into X coordinates it takes the form:

$$G_{IJ}(t) = \mathcal{V}^{-\frac{2}{3}} G_{IJ}(X) \tag{2.7}$$

with

$$G_{IJ}(X) = -\frac{1}{2} \left[C_{IJK} X^K - \frac{1}{4} (C_{IKL} X^K X^L)(C_{JMN} X^M X^N) \right] .$$

Notice, that $\frac{3}{2}G_{IJ}X^J = X_I$ is the normal vector and $\partial_A X^I$ are tangent vectors on the scalar manifold \mathcal{M} , as shown on Fig.1.

In order to perform the reduction of the $G \wedge^* G$ term in (2.1), consider the gauge field term (2.2) which in five dimensions becomes:

$$\int \sqrt{g_{str}} \mathcal{V} G_{IJ}(t) F_{\mu\nu}^I F_{\mu'\nu'}^J g_{str}^{\nu\nu'} g_{str}^{\mu\mu'} = \int \sqrt{g_E} G_{IJ}(X) F_{\mu\nu}^I F_{\mu'\nu'}^J g_E^{\nu\nu'} g_E^{\mu\mu'}$$

and (2.5) yields:

$$\int \sqrt{g_{str}} \mathcal{V} (dC_3)^2 = \int \sqrt{g_E} \mathcal{V}^2 (dC_3)^2 .$$

We are looking for potentials that we can obtain from non-trivial G -fluxes. The flux quantization condition can be written in the following form¹:

$$\int_Y G_{flux} \wedge \omega_I = \alpha_I = \text{integer} \quad (2.8)$$

Since the internal space remains a Calabi-Yau, in particular Ricci-flat, the only source for a potential comes from the G^2 term. The topological term contains a derivative in the uncompactified space and therefore cannot give a potential. Let us consider the example discussed in [8,9]:

$$G_{flux} = \frac{1}{\mathcal{V}} \alpha^I \star \omega_I \quad (2.9)$$

with $\alpha^I = G^{IJ}(t) \alpha_J$ in agreement with (2.8). This yields:

$$\int_{M_{11}} G_{flux} \wedge \star G_{flux} = 2 \int_{M_5} \sqrt{g_{str}} \frac{1}{\mathcal{V}} \alpha^I \alpha^J G_{IJ}(t) = 2 \int_{M_5} \sqrt{g_E} \frac{1}{\mathcal{V}^2} \alpha_I \alpha_J G^{IJ}(X)$$

Note the difference between $G_{IJ}(t)$ and $G_{IJ}(X)$, cf. (2.7). Writing the volume scalar as:

$$\mathcal{V} = e^{-2\varphi}$$

the potential becomes:

$$V(X, \varphi) = e^{4\varphi} \left(\alpha_I \alpha_J G^{IJ}(X) \right) \quad (2.10)$$

A potential of this form was originally found in [8].

If we include the G -flux in the topological term we obtain after compactification

$$\int_{M_{11}} G \wedge C \wedge G = \int_{M_5} G \wedge A^I \int_Y \omega_I \wedge^* \omega_J \alpha^J = \int_{M_5} G \wedge A^I \alpha_I \quad (2.11)$$

¹ In general, the periods α_I are only required to be half-integer [24].

and after dualization the 4-form G , the corresponding scalar a becomes charged under the gauge field $A^I \alpha_I$. This effect, as well as the generation of the potential (2.10) can also be understood from the reduction of the eleven-dimensional supersymmetry transformations:

$$\delta e_M^A = i\bar{\eta}\Gamma^A\psi_M, \quad \delta C_{MNP} = 3i\bar{\eta}\Gamma_{[MN}\psi_{P]}. \quad (2.12)$$

$$\delta\psi_M = \nabla_M\eta - \frac{1}{288}(\Gamma_M^{PQRS} - 8\delta_M^P\Gamma^{QRS})G_{PQRS}\eta. \quad (2.13)$$

Here the supersymmetry parameter η is an eleven-dimensional Majorana spinor. Under the split $11=5+6$, we decompose it as $\eta = \epsilon \otimes \xi$ where ϵ is an anti-commuting spinor in five non-compact dimensions.

If there were no background G -fluxes, then the resulting supersymmetry transformations in five dimensions would correspond to the usual (not gauged) supergravity theory which does not allow a scalar potential. This supergravity would have $SU(2)$ R -symmetry group. Gauging a $U(1)$ subgroup of the R -symmetry group, one obtains a gauged supergravity first found by Gunaydin, Sierra and Townsend [25]. They considered only vector multiplets which effectively means that the volume of the internal space is assumed to be fixed. In this case the supersymmetry transformations in the gauged theory differ only by the extra terms:

$$\delta\lambda^{iA} = P^A\delta^{ij}\epsilon_j \quad (2.14)$$

in the gaugino variation, and:

$$\delta\psi_\mu^i = \frac{i}{2\sqrt{6}}P_0\gamma_\mu\delta^{ij}\epsilon_j \quad (2.15)$$

in the variation of gravitino. In our notation $P^A \sim \partial^A W$ and $P_0 \sim W$. On the other hand, allowing for general G -fluxes also yields a dynamical volume scalar which is equivalent to a rescaling of W combined with an additional term in the potential V .

It is easy to see, for example, how (2.15) comes from the supersymmetry transformations (2.13) with a G -flux. If the background field G has non-zero components only in the internal space, then only the first term in brackets is relevant². The extra term (2.14) can be obtained in a similar way.

² In order to obtain (2.15), we also make a decomposition of gamma-matrices $\Gamma_\mu = \gamma_\mu \otimes \gamma_7$ and $\Gamma_m = 1 \otimes \gamma_m$ in the formula (2.13). The eleven-dimensional gamma-matrices Γ^M are hermitian for $M = 1, \dots, 10$ and anti-hermitian for $M = 0$.

To summarize, in the large volume limit the effective five-dimensional gauged supergravity action reads [8]:

$$S \sim \int \sqrt{G} \left[\frac{1}{2} R - g^2 V - \frac{1}{4} G_{IJ} F_{\mu\nu}^I F^{\mu\nu J} - \frac{1}{2} g_{AB} \partial_\mu \phi^A \partial^\mu \phi^B - \frac{1}{2} h_{rs} D_\mu q^r D^\mu q^s \right] + \int C_{IJK} F^I \wedge F^J \wedge A^K \quad (2.16)$$

with

$$g_{AB} = \partial_A X^I \partial_B X^J G_{IJ} \quad (2.17)$$

subject to the constraint $\frac{1}{6} C_{IJK} X^I X^K X^J = 1$. Using the convention of [8] the metric of the universal hypermultiplet h_{rs} is given by

$$h_{rs} dq^r dq^s = \frac{1}{4\mathcal{V}^2} d\mathcal{V}^2 + \frac{1}{2\mathcal{V}^2} \left[da + i(m d\bar{m} - \bar{m} dm) \right]^2 + \frac{1}{\mathcal{V}} dm d\bar{m} = u\bar{u} + v\bar{v} \quad (2.18)$$

where

$$u = \frac{dm}{\sqrt{\mathcal{V}}} \quad , \quad v = \frac{1}{2\mathcal{V}} (d\mathcal{V} + ida + m d\bar{m} - \bar{m} dm) \quad (2.19)$$

and this metric parameterizes the coset $SU(2,1)/U(2)$. Recall, \mathcal{V} is the volume scalar, the axionic scalar a comes from the dualization of the five-dimensional 3-form field and the complex scalar m was introduced in (2.6). Notice, due to the non-trivial flux only the axionic scalar a becomes charged:

$$D_\mu q^r = \{ \partial_\mu \mathcal{V} \ , \ \partial_\mu a + A_\mu^I \alpha_I \ , \ \partial_\mu m \ , \ \partial_\mu \bar{m} \} \quad (2.20)$$

In the supergravity theory this corresponds to a gauging of the axionic shift symmetry $a \rightarrow a + \text{const}$. In order to understand the structure of the potential, we have to understand the gauging on the supergravity side [8]. Obviously, the G -fluxes correspond to a gauging along the Killing vector:

$$k = \partial_a = \frac{i}{2\mathcal{V}} (\partial_v - \partial_{\bar{v}}) \quad (2.21)$$

The Killing prepotentials have the following form:

$$\mathcal{P}_I = \begin{pmatrix} -\frac{i}{4\mathcal{V}} \alpha_I & 0 \\ 0 & \frac{i}{4\mathcal{V}} \alpha_I \end{pmatrix} \quad (2.22)$$

and obey the relations:

$$k_I^u \mathcal{K}_{uv} = \nabla_s \mathcal{P}_I = \partial_v \mathcal{P}_I + [\omega_v, \mathcal{P}_I] \quad (2.23)$$

where ω_v is the v component of the $SU(2)$ connection and \mathcal{K}_{uv} is the triplet of Kähler forms³ $\mathcal{K}_{uv}^x = h_{uw}(J^x)_v^w$:

$$\omega = \begin{pmatrix} \frac{1}{4}(v - \bar{v}) & -u \\ u & -\frac{1}{4}(v - \bar{v}) \end{pmatrix}, \quad \mathcal{K} = \begin{pmatrix} \frac{1}{2}(u \wedge \bar{u} - v \wedge \bar{v}) & u \wedge \bar{v} \\ v \wedge \bar{u} & -\frac{1}{2}(u \wedge \bar{u} - v \wedge \bar{v}) \end{pmatrix} \quad (2.24)$$

The gauging fixes the potential to be of the following form:

$$\begin{aligned} V &= 4\text{tr}(\mathcal{P}_I \mathcal{P}_J) \left[2X^I X^J - G^{IJ} \right] + 2X^I X^J h_{uv} k_I^u k_J^v \\ &= 4e^{4\varphi} \left[g^{AB} \partial_A W \partial_B W - \frac{4}{3} W^2 \right] + 2e^{4\varphi} W^2 |k|^2 \end{aligned} \quad (2.25)$$

where

$$W \equiv \alpha_I X^I \quad (2.26)$$

Inserting this expression into (2.25), one finds that the last two terms in the first line cancel and the first term agrees with the potential in (2.10).

So far our discussion was purely classical. However, it is very easy to incorporate corrections due to the non-minimal terms in the action (2.1). It was found by Strominger [4] that corrections due to the terms proportional to the fourth power of the Riemann curvature simply lead to the shift (redefinition of the dilaton field):

$$e^{-2\varphi} \rightarrow e^{-2\varphi} + \frac{\chi(Y)}{15 \cdot 2^{10} \cdot \pi^8} \quad (2.27)$$

where $\chi(Y)$ is the Euler number of the Calabi-Yau space Y . In string theory this would be a one-loop correction to the metric on the moduli space of the universal hypermultiplet. We note that as long as we consider only the universal hypermultiplet, we do not expect further corrections especially no instanton corrections. In addition, the shift (2.27) effectively puts a lower bound on the “quantum” volume of the Calabi-Yau space, which leads to some qualitative changes of the supergravity solutions. As we will see in the section 4, the domain wall describes a supergravity solution with monotonically decreasing volume and if it eventually reaches this lower bound, we can keep the volume constant and allowing afterwards only internal deformations as described by the scalars in the vector multiplets. As consequence, at the point where this “quantum” volume is reached, the volume scalar effectively decouples from our supergravity solution and the scalars in the vector multiplets settle down at the extremum of the potential. This configuration is described by an AdS vacuum. Of course, this interpretation makes sense only if $\chi(Y) > 0$.

³ Here J^x denotes the triplet of complex structures.

3. More Superpotentials From Calabi-Yau Calibrations

In this section we discuss a way to derive the effective superpotentials via identification of BPS domain walls with branes wrapped over supersymmetric cycles. Although in this paper we are mainly interested in M theory compactifications on Calabi-Yau three-folds, we will also consider string theory compactifications.

Let us start with a general compactification of string theory or M theory on a compact oriented manifold Y to $(d + 1)$ non-compact dimensions. In other words, the (real) dimension of space Y is equal to $(9 - d)$ in string theory, or $(10 - d)$ in M theory. Trying to keep the discussion as general as possible, we make only a few minor assumptions about the geometry of the space-time. Namely, we assume that compactification on Y preserves some supersymmetry, so that it makes sense to talk about BPS domain walls in $(d+1)$ -dimensional effective theory. Non-compact space-time is assumed to be a maximally symmetric homogeneous space with zero or negative cosmological constant, *i.e.* Anti de Sitter space or a Minkowski space.

Assuming further the existence of a $(d + k - 1)$ -brane in the theory we start with, we can construct a BPS domain wall in the effective field theory by wrapping this brane over a supersymmetric k -cycle $\Sigma \subset Y$, of course, if there is one. Indeed, a simple counting of dimensions shows that the resulting object should be codimension one in the non-compact space-time. Notice, supersymmetric branes that we consider represent a magnetic source for some field strength in string theory or M theory, depending on the model in question. Let us call this field strength \mathcal{F} . Thus, as we move across the domain wall in $(d + 1)$ dimensions, the field strength jumps, $\mathcal{F} \rightarrow \mathcal{F} + \Delta\mathcal{F}$. The change of the flux, $\Delta\mathcal{F}$, is determined by the geometry of the $(d + k - 1)$ -brane that we used to construct the domain wall. Namely, we have:

$$\Delta\mathcal{F} = [\widehat{\Sigma}] \tag{3.1}$$

where the cohomology class $[\widehat{\Sigma}] \in H^*(Y, \mathbb{Z})$ is Poincaré dual to the homology class $[\Sigma]$.

Let us now return to the BPS property of the domain wall. Since BPS states have the least possible mass, and the domain wall in question is represented by a $(d + k - 1)$ -brane wrapped over k -dimensional cycle Σ , we conclude that Σ should have the minimal volume in its homology class. Due to this last property, calibrated geometries introduced by Harvey and Lawson [26] turn out to be very useful in a study of supersymmetric brane configurations (see [27] for a review and a list of references). Here we give only the definition of a calibrated submanifold and refer the reader to the original paper [26] for

further details. A closed k -form Ψ is called a calibration if its restriction to the tangent space $T_x\Sigma$ is not greater than the volume form of Σ for every submanifold $\Sigma \subset Y$. By saying this we mean that $\Psi|_{T_x\Sigma} \leq \text{vol}(T_x\Sigma)$ is satisfied provided that $\Psi|_{T_xS} = c \cdot \text{vol}(T_xS)$ for some real coefficient $c \leq 1$. Furthermore, if $\Psi|_{T_x\Sigma} = \text{vol}(T_x\Sigma)$ for every point $x \in \Sigma$, the submanifold is called a calibrated submanifold with respect to the calibration Ψ . It follows that calibrated submanifolds have the minimal volume in their homology class:

$$\text{Vol}(\Sigma) = \int_{\Sigma} \Psi \quad (3.2)$$

The last assumption we are going to make is that the mass of our BPS domain wall in the $(d+1)$ -dimensional effective theory is determined by the usual BPS formula:

$$M_{BPS} = |\Delta W| \quad (3.3)$$

where W is the effective superpotential. Then, combining the formulas (3.1), (3.2) and (3.3) together we obtain the following formula for the superpotential generated by a flux $\mathcal{F} \in H^*(Y)$:

$$W = \int_Y \Psi \quad (3.4)$$

The approach via calibrated geometries that we have outlined above can be applied to a computation of tree-level superpotentials induced by background fluxes in compactifications of string theory and M theory on Calabi-Yau manifolds [20,22,28], and to the derivation of membrane instanton superpotentials in M theory compactifications on G_2 manifolds [29]. Although all the results agree with what one finds studying the supersymmetry conditions, it would be also interesting to derive the effective superpotentials directly from the Kaluza-Klein reduction of the Lagrangian, *cf.* [28]. It is clear that in the case of Calabi-Yau four-folds, non-minimal terms like the anomaly term $C \wedge I_8(R)$ and, perhaps, their supersymmetric completion must play an important role [30].

Compactifications on $Spin(7)$ manifolds preserve only two real supercharges in the effective field theory. It was demonstrated in [31] that the BPS mass condition (3.3) is modified in such theories by the one-loop quantum anomaly $W \rightarrow W + \frac{W''}{4\pi}$. Therefore, one might expect that the effective superpotential induced by a four-form flux in compactification on $Spin(7)$ manifold is given by the appropriate modifications of the formula (3.4) which takes into account one-loop quantum anomaly. It would be interesting to see this anomaly by a direct computation of the superpotential via Kaluza-Klein reduction of the ten-dimensional supergravity action or supersymmetry transformations.

In this paper we focus on the case where Y is a Calabi-Yau three-fold. There are two types of calibrations on Calabi-Yau three-folds. The first type of calibrations — so-called Kähler calibrations — includes closed forms of even degree constructed from various powers of the Kähler form \mathcal{K} :

$$\Psi = \frac{1}{p!} \mathcal{K}^p \quad (3.5)$$

Apart from the trivial examples corresponding to $p = 0$ or $p = 3$, the submanifolds calibrated by such Ψ are holomorphic curves and divisors in Y . The second type of calibrations — the special Lagrangian calibration:

$$\Psi = \text{Re}(\Omega) \quad (3.6)$$

corresponds to special Lagrangian submanifolds in Y . Here $\Omega \in H^{3,0}(Y)$ is the unique holomorphic 3-form.

Clearly, the formula (3.4) can be used in compactifications of heterotic string theory on Calabi-Yau three-folds. In this case, four-dimensional effective field theory has $\mathcal{N} = 1$ supersymmetry. The only way to construct a BPS domain wall in four non-compact dimensions is to consider a five-brane wrapped around a special Lagrangian cycle in Y . Since a five-brane is a source for the Neveu-Schwarz three-form field strength, eq. (3.4) yields:

$$W = \int_Y H \wedge \Omega$$

We believe that this formula can be derived by the direct arguments, similar to what we used in the previous section.

It turns out that the formula (3.4) can be also applied to theories with larger supersymmetry. Recently, Taylor and Vafa [28] studied the effect of background fluxes in Type II string theory on (non-compact) Calabi-Yau three-folds. They found that it leads to partial supersymmetry breaking via generation of the effective superpotential (3.4) and reconciled it with the results of [17,18]. In particular, in Type IIA string theory on a Calabi-Yau three-fold Y the effective superpotential induced by the flux \mathcal{F} has the following form [28]:

$$W = \int_Y e^{\mathcal{K}} \wedge \mathcal{F}$$

which is exactly what follows from (3.4) with the Kähler calibration (3.5).

One goal of the present paper is to demonstrate that effective superpotential of the form (3.4) is also generated in compactification of M theory on a Calabi-Yau space Y

with a four-form field flux G . The resulting field theory in five dimensions has $\mathcal{N} = 2$ local supersymmetry. In $\mathcal{N} = 2$ five-dimensional gauged supergravity theories all our assumptions, including the BPS formula (3.3), are justified by the relation between central charge of $\mathcal{N} = 2$ supersymmetry algebra and the gravitino mass as discussed in [5,11]. Since the four-form field strength G is the only possible flux in M theory, the formula (3.4) predicts the following simple superpotential $W \sim \int_Y \mathcal{K} \wedge G_{flux} = \alpha_I t^I$. In the five-dimensional $\mathcal{N} = 2$ gauged supergravity theory we expect the effective superpotential W to be a function of the scalar fields X^I from vector multiplets, rather than t^I which also include a volume scalar \mathcal{V} from the universal hypermultiplet. Since α_I are integer numbers, after the appropriate rescaling we obtain the following superpotential:

$$W = \alpha_I X^I \tag{3.7}$$

which is nothing but the effective superpotential (2.26) found in the previous section via direct Kaluza-Klein reduction.

Notice that variation of the potential (3.7) with respect to the fields X^I leads to the condition:

$$G = 0$$

which means that there are no supersymmetric vacua in compactification of M theory on Calabi-Yau three-folds with non-trivial fluxes. In other words, the space of supersymmetric vacua has only one component corresponding to $\mathcal{F} = 0$, *cf.* (1.1).

4. Domain Wall Solutions

Motivated by [16], one may hope to understand non-perturbative effects in realistic models a la Randall-Sundrum via embedding the corresponding domain wall solutions in M theory or string theory. Since $\mathcal{N} = 2$ five-dimensional supergravity can be obtained from compactification of M theory on a Calabi-Yau three-fold Y , it is natural to assume that the domain wall is constructed out of M5-brane wrapped over a holomorphic curve $\Sigma \subset Y$, see also [8]. Then, topology of Y and Σ determine the spectrum of the low-energy theory on the five-brane, and the appropriate embedding $\Sigma \rightarrow Y$ may give us a theory close to the Standard Model. Note, because the curve Σ is holomorphic in Y , the effective four-dimensional theory has $\mathcal{N} = 1$ supersymmetry.

Interested in domain wall solutions in the five-dimensional supergravity we write the metric as:

$$ds^2 = e^{2U(y)} \left[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] + e^{-2\gamma U(y)} dy^2 \quad (4.1)$$

where the constant γ fixes the coordinate system and will be chosen later. This ansatz contains no restrictions as long as we regard the four-dimensional domain wall as a flat Minkowski space, but this parameterization will enable us to find an analytic solution below. Keeping the flat Minkowski space means also that the solution cannot carry electric and/or magnetic charges, but can carry a topological charge given by the difference of the cosmological constants. It is thus consistent to set all the gauge fields to zero. Moreover, investigating the equations of motion coming from the Lagrangian, we find that the complex scalar m and the axion a can be neglected because they do not show up in the potential. We will keep all scalars t^I , i.e. the scalars of in the vector multiplets ϕ^A and the volume scalar $\mathcal{V} = e^{-2\varphi}$.

4.1. Solution of the 5d Killing spinor equations

To ensure supersymmetry we have to solve the Killing spinor equations. Since the gauge fields are trivial for our domain wall the relevant variations are:

$$\begin{aligned} \delta\psi_\mu &= \left(\partial_\mu + \frac{1}{4} \omega_\mu^{ab} \Gamma_{ab} + \frac{1}{2} g \Gamma_\mu e^{2\varphi} W \right) \epsilon, \\ \delta\lambda_A &= \left(-\frac{i}{2} g_{AB} \Gamma^\mu \partial_\mu \phi^B + i \frac{3}{2} g e^{2\varphi} \partial_A W \right) \epsilon, \\ \delta\zeta &= e^{2\varphi} \left(-\frac{i}{2} \Gamma^\mu \partial_\mu e^{-2\varphi} - i 3 g X^I k_I \right) \epsilon \end{aligned} \quad (4.2)$$

with $W = \alpha_I X^I$. The scalar fields ϕ^A parameterize the manifold \mathcal{M} defined by (2.4), and the only non-trivial hypermultiplet field $e^{-2\varphi} = \mathcal{V}$ gives the Calabi-Yau volume.

Let us start with the gravitino variation $\delta\psi$. For our ansatz of the metric, the only non-zero components of the vielbeine and spin connection are:

$$e^m = e^U dx^m, \quad e^y = e^{-\gamma U} dy, \quad \omega^{my} = e^{(\gamma+1)U} U' dx^m \quad (4.3)$$

where $m = 0, 1, 2, 3$ and the corresponding gravitino variation becomes:

$$\delta\psi_m = \left(\frac{1}{2} e^{(\gamma+1)U} U' \Gamma_m \Gamma_y + \frac{1}{2} \Gamma_m g e^U e^{2\varphi} W \right) \epsilon \quad (4.4)$$

Using the projector $(1 + \Gamma_y)\epsilon = 0$ we find:

$$g e^{2\varphi} W = e^{\gamma U} U' . \quad (4.5)$$

From the $\delta\psi_y$ component:

$$0 = \delta\psi_y = \left(\partial_y + \frac{1}{2} e^{-\gamma U} \Gamma_y g e^{2\varphi} W \right) \epsilon \quad (4.6)$$

we obtain the Killing spinor after using (4.5):

$$\epsilon = e^{\frac{U}{2}} (1 - \Gamma_y) \epsilon_0 \quad (4.7)$$

where ϵ_0 is any constant spinor.

Moreover, using (4.5) we can also solve the hyperino variation:

$$\begin{aligned} 0 &= \left(-\frac{1}{2} \Gamma^\mu \partial_\mu e^{-2\varphi(y)} - 3gW \right) \epsilon \\ &= \left(\frac{1}{2} e^{\gamma U} (e^{-2\varphi})' - 3 e^{\gamma U} U' e^{-2\varphi} \right) \epsilon \\ &= \frac{1}{2} e^{\gamma U} e^{-2\varphi} (2\varphi' - 6 U') \epsilon \end{aligned} \quad (4.8)$$

and therefore

$$e^{6U} = e^{-2(\varphi - \varphi_0)} = \mathcal{V}/\ell \quad (4.9)$$

where $\ell = e^{-2\varphi_0}$ is the integration constant. Finally, we come to the gaugino variation $\delta\lambda_A$ which gives:

$$\begin{aligned} 0 &= -\frac{i}{2} \left(g_{AB} \Gamma^\mu \partial_\mu \phi^B - 3g e^{2\varphi} \partial_A W \right) \epsilon \\ &= -\frac{i}{2} \left(\Gamma^\mu \partial_A X^I \partial_B X^J G_{IJ} \partial_\mu \phi^B - 3g e^{2\varphi} \partial_A (\alpha_I X^I) \right) \epsilon \\ &= -\frac{i}{2} \partial_A X^I \left(e^{\gamma U} \frac{3}{2} \partial_y X_I - 3g e^{2\varphi} \alpha_I \right) \epsilon . \end{aligned} \quad (4.10)$$

Because $\partial_A X^I$ defines tangent vectors, the expression in brackets has to be proportional to the normal vector X_I :

$$\frac{3}{2} e^{\gamma U} (X_I)' - 3g e^{2\varphi} \alpha_I = -3e^{\gamma U} U' X_I = -\frac{3}{\gamma} (e^{\gamma U})' X_I \quad (4.11)$$

the coefficient on the rhs can be verified by contracting the equation with X^I and using (4.5). Next, replacing $e^{2\varphi}$ by employing (4.9) and taking $\gamma = -4$ we get

$$\frac{1}{2} \partial_y (e^{2U} X_I) = g \alpha_I / \ell \quad (4.12)$$

and thus the solution is

$$X_I \equiv \frac{1}{6} C_{IJK} X^J X^K = e^{-2U} \frac{1}{3} H_I = e^{-2U} \frac{1}{3\ell} (q_I + 6g\alpha_I y) \quad (4.13)$$

where q_I are arbitrary constants. This solution agrees with the one derived in [8], but notice also the close relationship to the attractor equations [32] which extremize the supersymmetry central charge, or in our case, the superpotential W and which state that at extrema of W the normal vector X_I becomes parallel to the flux vector α_I . These extrema are reached at $y \rightarrow \pm\infty$ where the scalars ϕ^A becomes constant and W extremal, due to (4.2). Remember, because of the run-away behavior of the volume, extrema of W are not extrema of the supergravity potential V .

As we discussed at the end of section 2, quantum corrections yield a lower bound for the volume, which is mainly given by the Euler number of the Calabi-Yau space. So, if we assume that in this “quantum” region the universal hypermultiplets effectively decouples and if we approximate the volume by the lower bound $\mathcal{V} = e^{-2\varphi_0} = \ell$, we find the same solution for eq. (4.11), but with $\gamma = +2$. In this case the spacetime metric becomes asymptotically anti de Sitter, which is expected because for a fixed volume, the potential has extrema.

4.2. Domain walls from five-branes in T^6

In the last section we showed that the Killing spinor equations are solved if the supergravity fields satisfy the eqs. (4.13) and (4.9) with the metric ansatz given by (4.1). Following [8], let us consider a simple example $Y = T^6$, where the intersection form is given by:

$$\frac{1}{6} C_{IJK} X^I X^J X^K = X^1 X^2 X^3 \quad (4.14)$$

For this example the equations (4.13) become:

$$\begin{aligned} H_1 &= e^{-2U} (X^2)(X^3) \\ H_2 &= e^{-2U} (X^1)(X^3) \\ H_3 &= e^{-2U} (X^1)(X^2) \end{aligned} \quad (4.15)$$

and thus:

$$X^1 = \frac{e^{2U}}{H_1} \quad , \quad X^2 = \frac{e^{2U}}{H_2} \quad , \quad X^3 = \frac{e^{2U}}{H_3} \quad , \quad e^{6U} = H_1 H_2 H_3 \quad (4.16)$$

For the generic case of a “running volume” ($\gamma = -4$) the domain wall metric reads:

$$ds^2 = (H_1 H_2 H_3)^{1/3} \left[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] + (H_1 H_2 H_3)^{4/3} dy^2 \quad (4.17)$$

and the volume is $\mathcal{V} = e^{6U} = H_1 H_2 H_3$ (setting $\ell = 1$). In order to understand the domain wall from the M theory perspective, we can rescale the solution and obtain for the string frame and for the Kähler class moduli:

$$\begin{aligned} ds_{str}^2 &= (H_1 H_2 H_3)^{-1/3} \left[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] + (H_1 H_2 H_3)^{2/3} dy^2 \\ t^I &= \mathcal{V}^{1/3} X^I = e^{2U} X^I = \frac{(H_1 H_2 H_3)^{2/3}}{H_I} \end{aligned} \quad (4.18)$$

In an infinite volume limit we can decompactify this solution and the 11-d metric becomes

$$ds_{11}^2 = \frac{1}{(H_1 H_2 H_3)^{1/3}} \left[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + (H_2 H_3 d\omega_1 + cycl.) + H_1 H_2 H_3 dy^2 \right] \quad (4.19)$$

where $d\omega_{1,2,3}$ are 2-d line elements and this configuration is an intersection $M5 \times M5 \times M5$ over a common 3-brane.

On the other hand in the fixed volume case, the X^I field are the same, but since $\gamma = 2$ the metric differs

$$ds^2 = (H_1 H_2 H_3)^{1/3} \left[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] + (H_1 H_2 H_3)^{-2/3} dy^2 \quad (4.20)$$

which yields AdS_5 for large y .

4.3. Discussion of some global aspects

Solving the local supergravity equations is not enough to describe domain walls, which are typically gravitational kink solutions that interpolate between vacua at $y = \pm\infty$. Interesting cases are interpolating solutions between vacua with different cosmological constants on both sides, i.e. the scalar fields flow between extrema of the potential. But there are also dilatonic domain walls, where the potential typically does not allow for isolated extrema and at least one scalar field “runs away”. This resembles the linear dilaton vacua appearing in certain string backgrounds. In the compactified theory, this run-away behavior signals a strong or weak coupling region, where the internal volume either diverges or shrinks. This is exactly the case for the solution that we described, where the volume of the internal space as described by the volume scalar φ diverges for $y \rightarrow +\infty$. On the other hand, the scalars in the vector multiplets are fixed by the attractor equations (4.13) and

become constant asymptotically, fixed only by the flux vector α_I . Therefore, the Killing spinor equations imply that asymptotically $\partial_A W = 0$ and we reach an extremum of the superpotential $W(\phi^A)$.

Note, the supergravity solution for the scalars $X^I(y)$ describes a trajectory on the (curved) moduli space and since it solves the equations of motion this trajectory is geodesic with radial coordinate y as affine parameter. This geodesic is fixed if we fix the two endpoints, i.e. two vacua. Let us stress, that it is not enough to fix only one endpoint, say, at $y = +\infty$, we have also to choose where the solution should flow at $y = -\infty$, i.e. on the other side of the wall.

But what happens if we pass the point $y = 0$? Our solution is valid for all values of y and the point $y = 0$ is generically not singular. So, we have to discuss the continuation to negative values of y . Because the domain wall is an intersection of five-branes, the flux-vector α_I should change at least the sign while passing the five-branes. As consequence, the product $\alpha_I y$ remains positive and we avoid singularities due to zeros of harmonic functions⁴ at finite values of y . A trivial possibility is to treat both sides symmetrically and therewith identifying both asymptotic vacua. More interesting, especially from the RG-flow point of view, is to patch together different vacua. An interesting case would be a solution interpolating between different vacua of a given superpotential W , but these domain walls are expected to be singular, because due to the global convexity of the moduli space [33] the attractor equations (4.13) have only one solution for a given Kaehler cone [34] and different extrema of W have to lie on disconnected branches of the moduli space. One can also consider a domain wall describing the flow towards vanishing volume. In this case the metric develops a singularity where the 4-dimensional world volume is squeezed to zero size, for examples see [35,11,36]. Let us comment on them in more detail.

The first thing to notice is that we can always approach this singular point by a proper choice of the vector q_I , which fixes the point $X^I(y = 0)$. A vanishing volume of the internal space yields always a singularity in supergravity solutions, but as we discussed earlier, quantum corrections or better higher curvature corrections provide a cut-off for the volume. This lower bound (2.27) was basically given by the Euler number of the internal manifold and therefore the regular supergravity solution can allow for at most 4 unbroken supercharges. By a simple shift in y we can always arrange that we reach this “quantum volume” at $y = 0$ and it is natural to describe the other side of the wall by the solution

⁴ Note, a vanishing harmonic function H_I means a vanishing cycle X_I .

with a fixed volume, i.e. $\gamma = 2$ in the solution described in section 4.1. Therefore in this regularized supergravity solution, the volume scalar flows from infinity (infinite volume) towards a lower bound and the scalars in the vector multiplets extremize on both sides the superpotential, i.e. they flow between fixpoints. Notice, the superpotential does not need to be the same on both sides, e.g. we may change the flux vector α_I on both sides and/or the intersection form but, due to the attractor equation combined with the convexity of \mathcal{M} , a given superpotential $W(\phi^A)$ has a unique extremum, where the flux vector α_I is parallel to the normal vector X_I (see Fig. 1). Moreover this type of domain wall provides an interesting example from AdS/CFT perspective, because the gauge theory couplings which are dual to the Kaehler class moduli t^I are UV-free, related on the sugra side to the infinite volume region, and flow in the IR to a non-trivial conformal fixpoints, where the supergravity solution becomes AdS_5 with fixed scalars.

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References

- [1] J.M. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity", Adv. Theor. Math. Phys. **2** (1998) 231; S.S. Gubser, I. R. Klebanov, A. M. Polyakov, "Gauge Theory Correlators from Non-Critical String Theory", Phys. Lett. **B428** (1998) 105; E. Witten, "Anti De Sitter Space And Holography", Adv. Theor. Math. Phys. **2** (1998) 253.
- [2] H.J. Boonstra, K. Skenderis, P.K. Townsend, "The domain-wall/QFT correspondence", JHEP **9901** (1999) 003.
- [3] K. Behrndt, E. Bergshoeff, R. Halbersma and J.P. van der Schaar, "On domain-wall/QFT dualities in various dimensions", Class.Quant.Grav. **16** (1999) 3317, hep-th/9907006.
- [4] A. Strominger, "Loop Corrections to the Universal Hypermultiplet", Phys. Lett. **B421** (1998) 139.
- [5] M. Cvetič, S. Griffies and Soo-Jong Rey, "Static domain walls in N=1 supergravity", Nucl.Phys. **B381** (1992) 301, hep-th/9201007. M. Cvetič and H.H. Soleng, "Supergravity domain walls", Phys.Rept. **282** (1997) 159, hep-th/9604090.
- [6] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, "Novel local CFT and exact results on perturbations of N = 4 super Yang-Mills from AdS dynamics", JHEP **12** (1998) 022, hep-th/9810126 .
- [7] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, "Renormalization group flows from holography supersymmetry and a c-theorem", hep-th/9904017.
- [8] A. Lukas, B.A. Ovrut, K.S. Stelle, D. Waldram, "The Universe as a Domain Wall", Phys.Rev. **D59** (1999) 086001; "Heterotic M-theory in Five Dimensions", Nucl.Phys. **B552** (1999) 246.
- [9] K.S. Stelle, "Domain Walls and the Universe", hep-th/9812086.
- [10] K. Behrndt, "Domain walls of D=5 supergravity and fixpoints of N=1 Super Yang Mills", hep-th/9907070.
- [11] K. Behrndt, M. Cvetič, "Supersymmetric Domain-Wall World from D=5 Simple Gauged Supergravity", hep-th/9909058.
- [12] M.J. Duff, "TASI Lecture on branes, black holes and anti-de Sitter space", hep-th/9912164.
- [13] M. Cvetič, H. Lü, C.N. Pope, "Domain walls and massive gauged supergravity potentials", hep-th/0001002
- [14] L. Randall and R. Sundrum, "An Alternative to Compactification", hep-th/9906064; L. Randall and R. Sundrum, "A Large Mass Hierarchy from a Small Extra Dimension", hep-ph/9905221.
- [15] R. Kallosh, A. Linde, "Supersymmetry and the Brane World", hep-th/0001071.
- [16] E. Witten, "Branes And The Dynamics Of QCD", Nucl.Phys. **B507** (1997) 658.

- [17] J. Polchinski and A. Strominger, ‘New Vacua for Type II String Theory,’ Phys. Lett. **B388** (1996) 736.
- [18] J. Michelson, “Compactifications of Type IIB Strings to Four Dimensions with Non-trivial Classical Potential”, Nucl. Phys. **B495** (1997) 127.
- [19] K. Becker, M. Becker, “M-Theory on Eight-Manifolds”, Nucl.Phys. **B477** (1996) 155.
- [20] S. Gukov, C. Vafa and E. Witten, “CFT’s From Calabi-Yau Four-folds”, hep-th/9906070.
- [21] K. Dasgupta, G. Rajesh, S. Sethi, “M Theory, Orientifolds and G-Flux”, hep-th/9908088.
- [22] S. Gukov, “Solitons, Superpotentials and Calibrations”, hep-th/9911011.
- [23] A.C. Cadavid, A. Ceresole, R. D’Auria, S. Ferrara, “11-Dimensional Supergravity Compactified on Calabi-Yau Threefolds”, Phys.Lett. **B357** (1995) 76.
- [24] E. Witten, “On Flux Quantization In M-Theory And The Effective Action”, J. Geom. Phys. **22** (1997) 1.
- [25] M. Gunaydin, G. Sierra, P.K. Townsend, “The N=2 Maxwell-Einstein Supergravity Theories: Their Compact and Noncompact Gaugings and Jordan Algebras”, Proc. of Nuffield Workshop (1985) 0367.
- [26] R. Harvey and H.B. Lawson, Jr., “Calibrated geometries”, Acta Math. **148** (1982) 47.
- [27] P.K. Townsend, “PhreMology–calibrating M-branes”, hep-th/9911154, contribution to *Strings ’99* proceedings.
- [28] T.R. Taylor, C. Vafa, “RR Flux on Calabi-Yau and Partial Supersymmetry Breaking”, hep-th/9912152.
- [29] R. Hernandez, “Calibrated Geometries and Non Perturbative Superpotentials in M-Theory”, hep-th/9912022.
- [30] S. Sethi, C. Vafa, and E. Witten, “Constraints on Low-Dimensional String Compactifications”, Nucl. Phys. **B480** (1996) 213.
- [31] M. Shifman, A. Vainshtein, M. Voloshin, “Anomaly and quantum corrections to solitons in two-dimensional theories with minimal supersymmetry”, Phys. Rev. **D59** (1999) 045016.
- [32] S. Ferrara, R. Kallosh, “Supersymmetry and attractors” Phys.Rev. **D54** (1996)1514, hep-th/9602136 , “Universality of supersymmetric attractors”, Phys.Rev. **D54** (1996) 1525, hep-th/9603090.
- [33] D. Morrison, “Beyond the Kaehler cone”, alg-geom/9407007.
- [34] M. Wijnholt, S. Zhukov, “On the Uniqueness of black hole attractors”, hep-th/9912002.
- [35] L. Giradello, M. Petrini, M. Porrati, A. Zaffaroni, “The supergravity dual of N=1 super Yang-Mills theory”, hep-th/9909047.
- [36] R. Kallosh, A. Linde, M. Shmakova, “Supersymmetric multiple basin attractors”, JHEP **9911** (1999) 010 , hep-th/9910021